1. Vypočtěte limity.

$$\lim_{[x,y]\to[0,0]} \frac{\sqrt{xy-1}-1}{x-y} \qquad \lim_{[x,y]\to[1,2]} \frac{1-xy}{1-xy}$$

$$\lim_{[x,y]\to[0,0]} \frac{xy}{x^2-y^2} \qquad \lim_{[x,y]\to[0,0]} \frac{x^2-y^2-xy^2}{x^2-y^2}$$

$$\lim_{[x,y]\to[0,0]} \frac{x^2}{x-y} \qquad \lim_{[x,y]\to[0,0]} \frac{x^2-y^2}{|x|-|y|}$$

$$\lim_{[x,y]\to[0,0]} \sqrt{x+\sqrt{y}} \qquad \lim_{[x,y]\to[0,0]} \frac{\sqrt{x-\sqrt{y}}}{xy}$$

$$\lim_{[x,y]\to[0,1]} \frac{\sqrt{x^2-(y-1)^2}-1}{x^2-(y-1)^2} \qquad \lim_{[x,y]\to[0,0]} \sqrt{\frac{x^2-y^2-x}{4x-x^2-y^2}}$$

$$\lim_{[x,y]\to[0,0]} \frac{e^{x^2+y^2}-1}{|x|-|y|} \qquad \lim_{[x,y]\to[0,0]} \frac{\ln(x-e^{x})}{\sqrt{x^2-y^2}}$$

$$\lim_{[x,y]\to[0,0]} \frac{x^2-y^2}{x^2-y^2} \qquad \lim_{[x,y]\to[0,0]} (x^2+y^2)^{x^2y^2}$$

$$\lim_{[x,y]\to[0,0]} (1+xy)^{\frac{1}{x+y}} \qquad \lim_{[x,y]\to[0,0]} \frac{1}{x^4-y^4} e^{-\frac{1}{x^2+y^2}}$$

$$\lim_{[x,y]\to[0,0]} (1+xy)^{\frac{1}{x+y}} \qquad \lim_{[x,y]\to[0,0]} \arctan\frac{y}{x}$$

2. Ukažte, že funkce f je řešením diferenciální rovnice r.

$f(x,y) = \arcsin\left(\frac{x-y}{x-y}\right)$ $\qquad r: \quad x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 0$

$f(t,x) = e^{-n^2kt}\sin nx \quad k \in R,\ n \in N$ $\qquad r: \quad \frac{\partial f}{\partial t} = k\frac{\partial^2 f}{\partial x^2}$

(rovnice vedení tepla)

$f(t,x,y) = e^{-(m^2-n^2)kt}\sin mx \cos ny \quad k \in R,\ m,n \in N$ $\qquad r: \quad \frac{\partial f}{\partial t} = k\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)$

(rovnice vedení tepla)

$f(t,x) = \cosh k(x-at) \quad k,a \in R$ $\qquad r: \quad a^2\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial t^2} = 0$

(rovnice struny)

$f(t,x) = \sin kx \cos \omega t \quad k,a \in R, \omega = ka$ $\qquad r: \quad a^2\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial t^2} = 0$

(rovnice struny)

$f(x,y) = \sqrt{x^2+y^2}$ $\qquad r: \quad f \cdot \Delta f = 0$

3. Vypočítejte derivaci f v bodě a ve směru vektoru $\vec{u}$ (podle vektoru $\vec{u}$).

$f(x,y)=2\sqrt{1-xy}+x+2y$	$a=[-1,2]$	$\vec{u}=(-1,3)$
$f(x,y)=\ln(1-x^2+y^2)$	$a=[1,e]$	$\vec{u}=(-1,e)$
$f(x,y)=\sqrt{x^2-y^2}+y\arcsin\frac{y}{x}$	$a=[2,\sqrt{2}]$	$\vec{u}=(1,1)$
$f(x,y)=(x-1)\sqrt{x-y^2}$	$a=[5,-2]$	$\vec{u}$ směr největšího stoupání, $\lvert\vec{u}\rvert=1$

4. Napište diferenciál funkce f v bodě a.

$f(x,y)=\frac{x^2-y^2}{x^2-y^2}$ $\qquad a=[1,2]$

$f(x,y)=\frac{xy}{z}e^{x^2-y^2-y^2}$ $\qquad a=[-1,2,3]$

$$f(x,y)=\begin{cases}0 & \text{pro [x,y]=[0,0]}\\ \frac{x^2}{x^2-y^2} & \text{jinak}\end{cases} \qquad a=[0,0]$$

$$f(x,y)=\begin{cases}0 & \text{pro [x,y]=[0,0]}\\ xy\frac{x^2-y^2}{x^2-y^2} & \text{jinak}\end{cases} \qquad a=[0,0]$$

diferenciál druhého řádu $f(x,y)=3x^2y-6xy+y^3$ $\qquad a=[2,1]$

5. Zjistěte tečnou rovinu a normálu k funkci f
a) v bodě T (dopočítejte chybějící souřadnici),

$f(x,y)=x^2+4y^2$	$T=[3,-1,.]$
$f(x,y)=\sqrt{x^2+y^2}-xy$	$T=[3,4,.]$
$f(x,y)=x^4-2x^2y+xy+2y$	$T=[1,.,3]$
$f(x,y)=\ln(xy^2)+x^2y$	$T=[\frac{1}{4},2,.]$
$f(x,y)=e^{xy}$	$T=[1,2,.]$
$f(x,y)=x^2-y^2$	$T=[1,1,.]$
$f(x,y)=\arctan\frac{x}{y}$	$T=[1,1,.]$
$f(x,y)=e^{-(x^2-y^2)}$	$T=[1,2,.]$
$f(x,y)=\sqrt{x-\sqrt{y}}$	$T=[.,1,2]$
$y=f(x,y);\quad x+y^2+z^2=18$	$T_1=[.,3,2]$ a $T_2=[.,-3,-2]$

b) rovnoběžnou s rovinou ρ.

$$f(x,y) = 2x^2 - y^2 \qquad \rho: \quad 8x - 6y - z - 15 = 0$$

$$z = f(x,y); \quad x^2 + 2y^2 + z^2 = 1 \qquad \rho: \quad 4x + 2y + z = 0$$

$$z = f(x,y); \quad z^2 = xy \qquad \rho: \quad 4x + y - 4z + 5 = 0$$

c) Zjistěte tečnu a normálu k funkci $y = f(x)$; $\quad 9e^{x-y} - x^2 + \sqrt{y} - \sqrt{3} = 0$ v bodě $T = [-3, 3]$.

d) Zjistěte úhel stoupání funkce $f(x,y) = e^{x^2-y^2}(x^2 + 2y)$ v bodu $T = [1, 2, .]$ ve směru $\vec{s} = (1, -1)$.

e) Zjistěte směr a úhel největšího stoupání funkce $f(x,y) = e^{x^2-y^2}(x^2 + 2y)$ v bodu $T = [1, 2, .]$.

6. Vyšetřete extrémy funkce f.

$$f(x,y) = x^2 + y^2 + 2x + 1 \qquad f(x,y) = xy + x + y - x^2 - y^2 + 2$$

$$f(x,y) = (x - 2y + 1)^4 \qquad f(x,y) = x^3 + y^3 - 12xy + 215$$

$$f(x,y) = x^3y(6 - x - y) \qquad f(x,y) = x\sqrt{y} - x^2 - 2y + 21x - 26$$

$$f(x,y) = 5xy + \frac{25}{x} + \frac{8}{y} \qquad f(x,y) = x^4 + y^4 - x^2 - 2xy - y^2$$

$$f(x,y) = xy \ln(x^2 + y^2) \qquad f(x,y) = e^{x-y^2}(x - 2y + 5)$$

$$f(x,y) = x + y + 4 \sin x \cos x \qquad f(x,y) = 2x^3 - xy^2 + 5x^2 + y^2$$

$$f(x,y) = \ln(x - y) - 2x^2 - 4y \qquad f(x,y) = e^{4y}(x^2 + 2y)$$

$$f(x,y) = 3x + 6y - x^2 - xy - y^2 \qquad f(x,y) = e^{-(x^2-y^2)}(2y^2 + x^2)$$

$$z = f(x,y); \quad x^2 + y^2 + z^2 - 2x + 2y - 4z - 10 = 0$$

$$z = f(x,y); \quad x^2 - y^2 + z^2 - 3x + 4y + z - 8 = 0$$

7. a) Vyšetřete extrémy funkce f na množině popsané rovnicí $g(x,y) = 0$.

$$f(x,y) = 4x^2 + y^2 \qquad g(x,y) = x^2 + y - 11$$

$$f(x,y) = xy \qquad g(x,y) = x^2 + y - 12$$

$$f(x,y) = 4x^3 + 3x^2 \qquad g(x,y) = x^2 + y - 12$$

$$f(x,y) = y^2 + 24x^2 + x^3 - x^4 \qquad g(x,y) = x^2 + y - 12$$

$$f(x,y) = x^2 + y^2 \qquad g(x,y) = x^3 + y^3 - 3xy$$

$$f(x,y) = x + y + 2 \qquad g(x,y) = 2(x^2 + y^2) - x^2y^2$$

b) Zjistěte bod křivky $x^2 - y^2 = 4$ nejbližší bodu $T = [0, 2]$.

8. Vypočítejte div $\vec{f}(A)$ a rot $\vec{f}(A)$.

$\vec{f}(x,y,z) = (y+z, x+z, x+y)$ $\qquad A = [\sqrt{2}, \pi, e]$

$\vec{f}(x,y,z) = (3x^3yz, 2xy^2z, xyz)$ $\qquad A = [-1, 2, 1]$

$\vec{f}(x,y,z) = (\sin(y+z), \sin(x+z), \sin(x+y))$ $\qquad A = [0, \frac{\pi}{2}, 0]$

$\vec{f}(x,y,z) = (yz\sqrt{3+x^2}, xz\sqrt{y^2-5}, xy\sqrt{z^2+5})$ $\qquad A = [1, -3, 2]$

$\vec{f}(x,y,z) = (x\ln(y+z), 3y\ln(x+z), 2z\ln(x+y))$ $\qquad A = [0, 2, 3]$

$\vec{f}(x,y,z) = (16z\sqrt{\frac{x}{y}}, 6\frac{\sqrt{xy}}{z}, -xy\sqrt{z})$ $\qquad A = [9, 4, 1]$

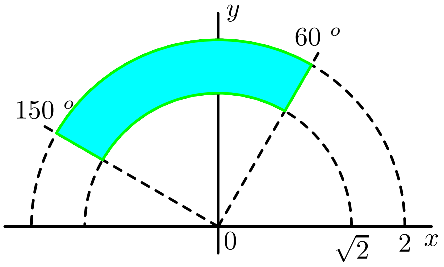
$\vec{f}(x,y,z) = (e^{x^2}, e^{y^2}, e^{z^2})$ $\qquad A = [1, 0, 1]$

$\vec{f}(x,y,z) = (e^{\frac{1}{z}}, e^{\frac{z}{x}}, e^{\frac{x}{y}})$ $\qquad A = [1, 1, 1]$

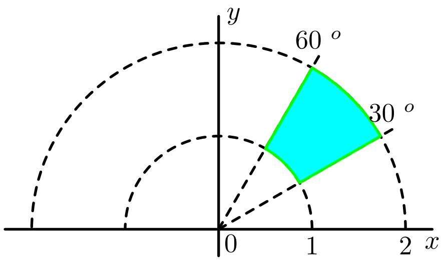
$\vec{f}(x,y,z) = (\cos y + \sin z), \cos z + \sin x, \cos x + \sin y)$ $\qquad A = [0, \frac{\pi}{2}, \pi]$

$\vec{f}(x,y,z) = (\ln x^2y, \ln y^2z, \ln z^2x)$ $\qquad A = [1, 2, 1]$

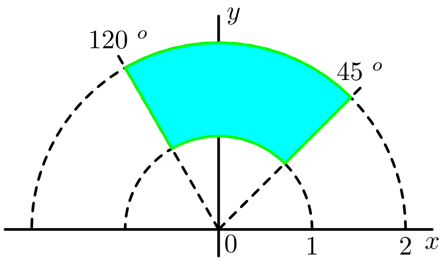
9. Vypočítejte dvojný integrál.



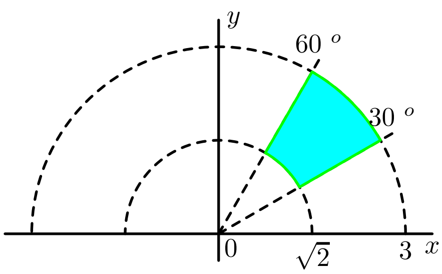
$$\iint_\Omega \frac{y}{x}\, dx\, dy$$



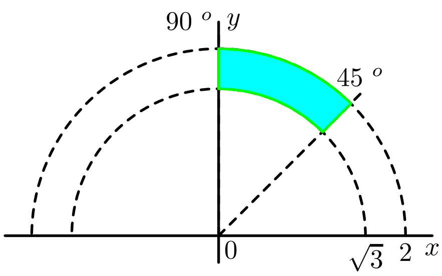
$$\iint_{\Omega} xy^2\,dx\,dy$$



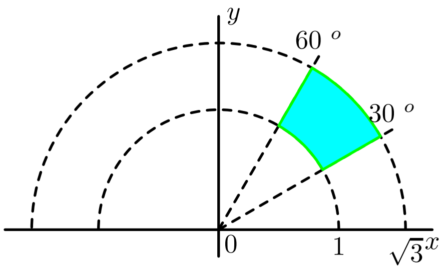
$$\iint_{\Omega} x^2y\,dx\,dy$$



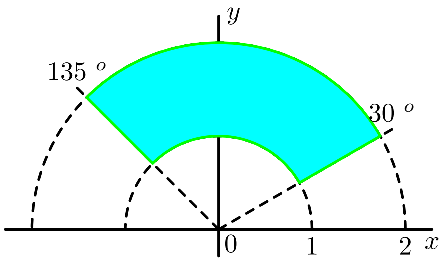
$$\iint_{\Omega} \frac{y}{x^2}\,dx\,dy$$



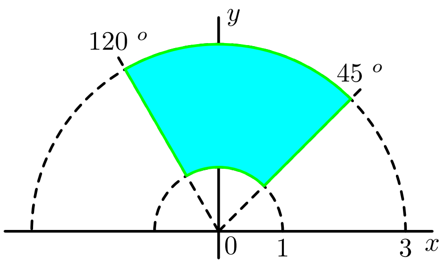
$$\iint_{\Omega} \frac{x}{y^2}\,dx\,dy$$



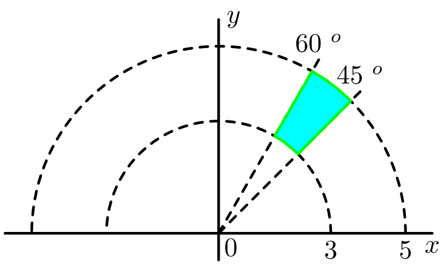
$$\iint_{\Omega} \frac{y}{1x^2}\,dx\,dy$$



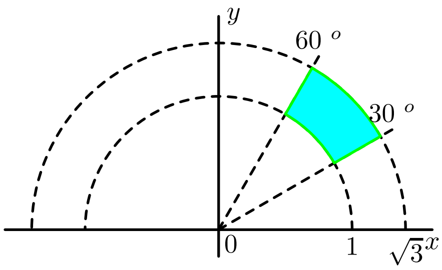
$$\iint_{\Omega} \frac{x}{y}\,dx\,dy$$



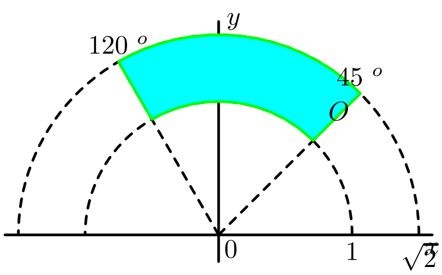
$$\iint_{\Omega} xy\,dx\,dy$$



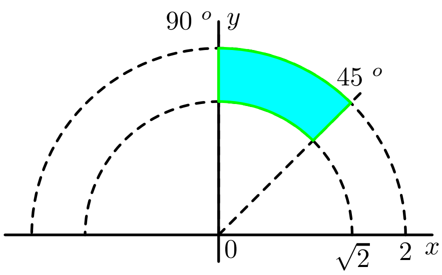
$$\iint_{\Omega} \frac{y}{x}\,dx\,dy$$



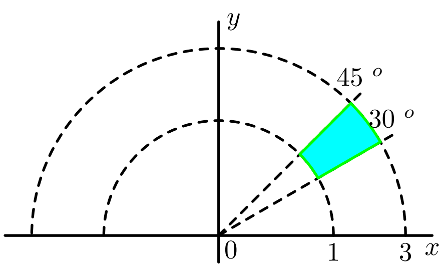
$$\iint_\Omega xy^2\,dx\,dy$$



$$\iint_\Omega x^2y\,dx\,dy$$



$$\iint_\Omega \frac{x}{y^2}\,dx\,dy$$



$$\iint_\Omega \frac{x}{y}\,dx\,dy$$

10. Vypočítejte objem

a) čtyřstěnu $ABCD$,

$$A = [0, 2, 3],\ B = [1, 0, 2],\ C = [2, -3, 4],\ D = [4, -1, 2]$$

$$A = [1, -2, 3],\ B = [0, 1, 1],\ C = [5, 3, 2],\ D = [3, 0, 1]$$

$$A = [2, 1, 2],\ B = [1, -1, 0],\ C = [1, 2, -1],\ D = [3, 0, -3]$$

b) tělesa Ω.

$$\Omega = \left\{[x, y, z]; 0 \le x \le 1, |y| \le x, 0 \le z \le x^2 y\right\}$$

$$\Omega = \left\{[x, y, z]; 0 \le y \le 1, x^2 \le y, 0 \le z \le x^2 + y\right\}$$

$$\Omega = \left\{[x, y, z]; 0 \le y \le 1, 0 \le x \le \sqrt{y}, 0 \le z \le x + y^2\right\}$$

$$\Omega = \left\{[x, y, z]; 0 \le y \le 2, 0 \le 2x \le y, 0 \le z \le y^2 - x^2\right\}$$

$$\Omega = \left\{[x, y, z]; 0 \le x \le 2, y \le x \le 2y, 0 \le z \le xy^2\right\}$$

$$\Omega = \left\{[x, y, z]; 0 \le x \le 3y, 0 \le y \le 1, 0 \le z \le x^2 + y^2\right\}$$

$$\Omega = \left\{[x, y, z]; 0 \le y \le 1, 0 \le x \le \arctan y, 0 \le z \le \frac{6x}{1-y^2}\right\}$$

$$\Omega = \left\{[x, y, z]; 1 \le x \le 2, 0 \le x \le \ln y, 0 \le z \le \frac{6x}{y}\right\}$$

$$\Omega = \left\{[x, y, z]; 0 \le y \le 1, 0 \le x \le \sqrt{1-y}, 0 \le z \le 4xy\right\}$$

11. Vypočítejte křivkový integrál

$$\int_\gamma h(x, y)\, ds \qquad \text{nebo} \qquad \int_\gamma \vec{f}(x, y)\, d\vec{r}\,.$$

γ: $x = \sqrt{1+t}$, $y = \sqrt{1-t}$, $t \in \langle 0, \frac{1}{2} \rangle$, $h(x, y) = \frac{1}{xy}$, $\vec{f}(x, y) = (x, y)$

γ: $x = \sqrt{1+t}$, $y = \sqrt{1-t}$, $t \in \langle 0, 1 \rangle$, $h(x, y) = xy$, $\vec{f}(x, y) = (y, x)$

γ: $x = t + \cos t$, $y = \sin t$, $t \in \langle 0, \frac{\pi}{2} \rangle$, $h(x, y) = \sqrt{1+y}$

γ: $x = a\cos t$, $y = a\sin t$, $t \in \langle 0, \frac{\pi}{2} \rangle$, $h(x, y) = x^2$

γ: $x = \cos t$, $y = \sin t$, $t \in \langle 0, \pi \rangle$, $h(x, y) = \sqrt{1 + x^2 + y^2}$, $\vec{f}(x, y) = (x, y)$

$\gamma:\quad \begin{array}{l} x=\frac{1}{2}\cos 2t \\ y=\sin t \end{array} \quad t\in(0,\frac{\pi}{2}) \qquad h(x,y)=\frac{1}{4y^2-1}$

$\gamma:\quad \begin{array}{l} x=t^2 \\ y=\ln t \end{array} \quad t\in(1,2) \qquad h(x,y)=\frac{4xy}{\sqrt{1-4x^2}} \qquad \vec{f}(x,y)=(2y-\ln x, x^2)$

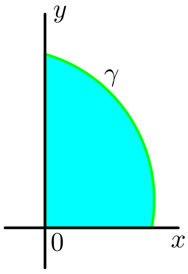
$\gamma:\quad \begin{array}{l} x=\frac{1}{t} \\ y=\frac{1}{2}t^2 \end{array} \quad t\in(1,2) \qquad h(x,y)=\frac{1}{x\sqrt{1-6y^2}} \qquad \vec{f}(x,y)=(2y, x^2)$

$\gamma:\quad \begin{array}{l} x=\frac{1}{t} \\ y=t \end{array} \quad t\in(1,2) \qquad \vec{f}(x,y)=(y^2, x^2)$

$\gamma:\quad \begin{array}{l} x=t\cos t \\ y=t\sin t \end{array} \quad t\in(0,2\pi) \qquad h(x,y)=3\sqrt{x^2+y^2} \qquad \vec{f}(x,y)=(x,y)$

γ : trojúhelník ABC, kde $A=[1,0,0]$, $B=[0,1,0]$, $C=[0,0,1]$ $\qquad \vec{f}(x,y,z)=(y^2,z^2,x^2)$

12. Vypočítejte obsah rovinného útvaru ohraničeného křivkou γ, osou x a y.



$\gamma:\quad \begin{array}{l} x=(t+\pi)\cos t \\ y=(t+\pi)\sin t \end{array} \quad t\in(0,\frac{\pi}{2})$

$\gamma:\quad \begin{array}{l} x=\sqrt{\pi^2-t^2}\cos t \\ y=\sqrt{\pi^2-t^2}\sin t \end{array} \quad t\in(0,\frac{\pi}{2})$

$\gamma:\quad \begin{array}{l} x=\frac{\cos t}{1-t^2} \\ y=\frac{\sin t}{1-t^2} \end{array} \quad t\in(0,\frac{\pi}{2})$

$\gamma:\quad \begin{array}{l} x=\sqrt{2t+\pi}\cos t \\ y=\sqrt{2t+\pi}\sin t \end{array} \quad t\in(0,\frac{\pi}{2})$

$\gamma:\quad \begin{array}{l} x=\frac{\cos t}{t-\frac{\pi}{2}} \\ y=\frac{\sin t}{t-\frac{\pi}{2}} \end{array} \quad t\in(0,\frac{\pi}{2})$

$\gamma:\quad \begin{array}{l} x=e^{-t}\cos t \\ y=e^{-t}\sin t \end{array} \quad t\in(0,\frac{\pi}{2})$

13. Vypočítejte plošný integrál

$$\iint_{\sigma} h(x,y,z)\,dS \qquad \text{nebo} \qquad \int_{\sigma} \vec{f}(x,y,z)\,d\vec{S}\,.$$

$\kappa: \quad x = u\cos v,\ y = u\sin v,\ z = u^2$	$u \in (0,3)$, $v \in (0, \frac{\pi}{2})$	$h(x,y,z) = 2$	
$\kappa: \quad x = u\cos v,\ y = u\sin v,\ z = u^2$	$u \in (0,1)$, $v \in (0, \frac{\pi}{2})$	$h(x,y,z) = \frac{xy}{\sqrt{4z-1}}2$	$\vec{f}(x,y,z) = (-y, x, xyz)$
$\kappa: \quad x = u\cos v,\ y = u\sin v,\ z = u^2$	$u \in (0,2)$, $v \in (0, \frac{\pi}{2})$	$h(x,y,z) = \sqrt{4z+1}$	$\vec{f}(x,y,z) = (x, y, z)$
$\kappa: \quad x = u\cos v,\ y = u\sin v,\ z = \sqrt{u}$	$u \in (0,1)$, $v \in (0, \frac{\pi}{2})$	$h(x,y,z) = 6(x^2+y^2)z$	$\vec{f}(x,y,z) = (y, -x, z^2)$
$\kappa: \quad x = u+2,\ y = v-2,\ z = uv-4$	$u \in (0,2)$, $v \in (0,1)$	$h(x,y,z) = \frac{z^2}{\sqrt{(x-y)^2-2z-7}}$	
$\kappa: \quad x = \frac{1}{2}u\cos v,\ y = \frac{1}{2}u\sin v,\ z = \frac{\sqrt{3}}{2}u$	$u \in (0,2)$, $v \in (0, \frac{\pi}{2})$	$h(x,y,z) = xz$	
$\kappa: \quad x = 2\cos u\sin v,\ y = 2\sin u\sin v,\ z = 2\cos v$	$u, v \in (0, \frac{\pi}{2})$	$h(x,y,z) = xz$	
$\kappa: \quad x = \frac{\sqrt{2}}{2}u\cos v,\ y = \frac{\sqrt{2}}{2}u\sin v,\ z = \frac{\sqrt{2}}{2}u$	$u \in (0,3)$, $v \in (0, 2\pi)$		$\vec{f}(x,y,z) = (y-z, z-x, x-y)$
$\kappa: \quad x = u+v,\ y = u-v,\ z = uv$	$u, v \in (0,1)$	$h(x,y,z) = \frac{z}{\sqrt{y^2-2x-2}}$	$\vec{f}(x,y,z) = (y, x, z)$
$\kappa:$ povrch čtyřstěnu $x=0, y=0, z=0, x+y+z=1$			$\vec{f}(x,y,z) = (xz, xy, yz)$
$\kappa: \quad x = \frac{u}{v},\ y = uv,\ z = u^2+v^2$	$u \in (0,1)$, $v \in (1,2)$		$\vec{f}(x,y,z) = (x^2, -xy, z^2)$
$\kappa: \quad x = u^2-v,\ y = u-v,\ z = uv$	$u, v \in (0,1)$		$\vec{f}(x,y,z) = (x, y, z)$
$\kappa: \quad x = \frac{u}{v},\ y = \frac{v}{u},\ z = uv$	$u, v \in (1,2)$	$h(x,y,z) = \frac{z}{\sqrt{x^2-y^2}}$	
$\kappa: \quad x = u^2,\ y = v^2,\ z = uv$	$u, v \in (1,3)$	$h(x,y,z) = \frac{z}{\sqrt{x^2-y^2-4z^2}}$	