Decide, whether the following statements are true or not and state the reason or the counterexample.

$$(\forall n \in \mathbf{N})\, a_n < b_n \Longrightarrow \lim_{n\to\infty} a_n < \lim_{n\to\infty} b_n$$

$$(a_n \to 0 \text{ and } a_n \cdot b_n \to 0) \Longrightarrow b_n \text{ bounded}$$

$$(a_n \to \infty \text{ and } a_n \cdot b_n \to 0) \Longrightarrow b_n \text{ bounded}$$

$$(a_n \to \infty \text{ and } a_n \cdot b_n \to \infty) \Longrightarrow a_n \text{ bounded}$$

$$a_n \to 0 \Longrightarrow \tfrac{1}{a_n} \to \infty$$

$$\sin a_n \to 0 \Longrightarrow a_n \to 0$$

$$\ln a_n \to 1 \Longrightarrow a_n \to e$$

$$a_n \to \infty \Longrightarrow a_n \text{ is not bounded}$$

$$a_n \text{ increasing } \Longrightarrow a_n \text{ is not decreasing}$$

$$a_n \text{ is not increasing } \Longrightarrow a_n \text{ decreasing}$$

$$\left(\tfrac{a_n}{b_n} \to 2 \text{ and } b_n \to \infty\right) \Longrightarrow a_n \to \infty$$

$$a_n \text{ increasing } \Longrightarrow a_n \text{ convergent}$$

$$a_n \text{ both increasing and bounded } \Longrightarrow a_n \text{ convergent}$$

$$a_n \to -\infty \Longrightarrow a_n \text{ is bounded above}$$

$$a_n \text{ convergent } \Longleftrightarrow a_n \text{ bounded}$$

$$a_n \text{ divergent } \Longrightarrow a_n \text{ is not bounded}$$

$$a_n \text{ is not bounded } \Longrightarrow a_n \text{ divergent}$$

$$a_n \text{ strictly increasing } \Longrightarrow a_n \to \infty$$

$$\sqrt{a_n} \to \infty \Longrightarrow a_n \to \infty$$

$$a_n^2 \to \infty \Longrightarrow a_n \to \infty$$

$$(\forall n \in \mathbf{N})\, a_n \geq n^2 \Longrightarrow \tfrac{n}{a_n} \to 0$$

$$\limsup_{n\to\infty} a_n = 1 \Longrightarrow a_n \text{ bounded}$$

$$(\forall n \in \mathbf{N})\, \sqrt{a_n} \leq \tfrac{1}{2} \Longrightarrow a_n \text{ convergent}$$

$$(\forall n \in \mathbf{N})\, a_{n+1} - a_n = \tfrac{1}{2^n} \Longrightarrow a_n \text{ convergent}$$

$$\limsup_{n\to\infty} a_n \leq \liminf_{n\to\infty} a_n \Longrightarrow a_n \text{ bounded}$$

$$(a_n \to \infty \text{ and } b_n \to \infty) \Longrightarrow a_n - b_n \to \infty$$

$$(a_n \to 0 \text{ and } b_n \to -\infty \text{ and } (\forall n \in \mathbf{N})\ b_n \neq 0) \Longrightarrow \tfrac{a_n}{b_n} \to 0$$

$$\left(f : [1;\infty[\to \mathbf{R} \text{ and } \lim_{x\to\infty} f(x) \text{ does not exist }\right) \Longrightarrow \text{ limit of sequence } \{f(n)\}_{n=1}^{\infty} \text{ does not exist}$$

$$(a_n \to \infty \text{ and } b_n \to 0 \text{ and } (\forall n \in \mathbf{N})\ b_n \neq 0) \Longrightarrow \tfrac{a_n}{b_n} \to \infty$$

$$(a_n \to 0 \text{ and limit of } b_n \text{ does not exist }) \Longrightarrow \text{ limit of } a_n + b_n \text{ does not exist}$$

$$(a_n \to 0 \text{ and limit of } b_n \text{ does not exist }) \Longrightarrow \text{ limit of } a_n \cdot b_n \text{ does not exist}$$

$$(a_n \to \infty \text{ and } b_n \to -\infty \text{ and } (\forall n \in \mathbf{N})\ b_n \neq 0) \Longrightarrow \tfrac{a_n}{b_n} \to 0$$

$$(a_n \to 1 \text{ and } b_n \to 0 \text{ and } (\forall n \in \mathbf{N})\ a_n > 0) \Longrightarrow a_n^{b_n} \to 1$$

$$(a_n \to 0 \text{ and } b_n \to 0 \text{ and } (\forall n \in \mathbf{N})\ a_n > 0) \Longrightarrow a_n^{b_n} \to 0$$

$$(a_n \to \infty \text{ and } b_n \to 0 \text{ and } (\forall n \in \mathbf{N})\ a_n > 0) \Longrightarrow a_n^{b_n} \to 1$$

$$(\ln a_n \to 1 \text{ and } b_n \to 1) \Longrightarrow a_n^{b_n} \to e$$

$$(a_n \to 1 \text{ and } \ln b_n \to 1 \text{ and } (\forall n \in \mathbf{N})\ a_n > 0) \Longrightarrow a_n^{b_n} \to 1$$

$$(\text{ limit of } a_n \text{ does not exist and limit of } b_n \text{ does not exis }) \Longrightarrow \text{ limit of } a_n \cdot b_n \text{ does not exis}$$

$$\left(a_n \to a \text{ and } \lim_{x\to a} f(x) = A\right) \Longrightarrow f(a_n) \to A$$

$$a_n \to a \Longrightarrow (\forall \epsilon > 0)(\exists n \in \mathbf{N})\, |a - a_n| < \epsilon$$

$$a_n \to a \Longleftrightarrow (\forall \epsilon > 0)(\exists n \in \mathbf{N})\, |a - a_n| < \epsilon$$

$$a \in \mathbf{R} \text{ is accumulation point of } a_n \Longleftrightarrow (\forall \epsilon > 0)(\exists n \in \mathbf{N})\, |a - a_n| < \epsilon$$

$$(\forall K \in \mathbf{R})(\exists n \in \mathbf{N})\, K < a_n \Longrightarrow a_n \to \infty$$

$$(\forall K \in \mathbf{R})(\exists n \in \mathbf{N})\, K < a_n \Longrightarrow \liminf_{n\to\infty} a_n = \infty$$

$$(\forall K \in \mathbf{R})(\exists n \in \mathbf{N})\, K < a_n \Longleftrightarrow \limsup_{n\to\infty} a_n = \infty$$

$$\lim_{n\to\infty} a_n < \lim_{n\to\infty} b_n \Longrightarrow (\exists n_0 \in \mathbf{N})(\forall n \geq n_0)\, a_n < b_n$$

$$(\exists A,\ B \text{ accumulation point of } a_n)\, A \neq B \Longrightarrow \limsup_{n\to\infty} a_n > \liminf_{n\to\infty} a_n$$

$$\left(\{a_{p(n)}\}_{n=1}^{\infty} \text{ subsequence of } \{a_n\}_{n=1}^{\infty} \text{ and } \limsup_{n\to\infty} a_{p(n)} = 2\right) \Longrightarrow 2 \text{ is accumulation point of } a_n$$

$$\left(\{a_{p(n)}\}_{n=1}^{\infty} \text{ subsequence of } \{a_n\}_{n=1}^{\infty} \text{ and } \lim_{n\to\infty} a_n = 3\right) \Longrightarrow \limsup_{n\to\infty} a_{p(n)} \geq 3$$

$$\{a_2, a_1, a_4, a_3, a_6, a_5, \dots\} \text{ is subsequence of } \{a_n\}_{n=1}^{\infty}$$

$$(\forall \{a_{p(n)}\}_{n=1}^{\infty} \text{ subsequence of } \{a_n\}_{n=1}^{\infty})\ \lim_{n\to\infty} a_{p(n)} = 4 \Longrightarrow \lim_{n\to\infty} a_n = 4$$

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